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A HARDNESS LAW FOR INELASTIC DEFORMATION

by

S. R. BODNER

MML Report No. 52

November 1976

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הפקולטה להנדסת מכונות

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A HARDNESS LAW FOR INELASTIC DEFORMATION

by

S. R. Bodner[1]

Abstract

Certain formulations of elastic-viscoplastic constitutive equations lead to inelastic deformations being governed by a functional relation between the plastic deformation rate and the stress. This relation is taken to be a unique, continuous function at a constant microstructural condition specified by certain state variables. On the assumption that a single state variable referred to as "hardness" is adequate to represent overall resistance to plastic flow, the problem becomes one of determining evolutionary equations for the dependence of the hardness parameter on loading history. A procedure for calculating hardness changes for general multiaxial loading histories is proposed in this paper. Stress-strain relations that would be predicted by this procedure appear to be consistent with experimental results for various multiaxial loading histories including proportional and non-proportional cyclic loading.

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Introduction

Attempts to represent inelastic behavior of materials more realistically than the classical idealizations have led to constitutive equations in which elastic and inelastic deformation rate components are functions of state variables and the current geometry. Examples of such incremental formulations of elastic-viscoplastic material response are Partom (1967), Bodner (1968), Bodner and Partom (1972 a,b; 1975; 1976), Rice (1970, 1975), Kelly and Gillis (1974), Hart (1975), and Miller (1976). An assumption in most of these theories is that both deformation rate components are always non-zero and that the same equations should hold for all loading conditions. A yield criterion is therefore not specified in the sense of a discontinuity condition between elastic and inelastic behavior and the stress-strain curve at a given strain rate is the solution of the equations for the particular case of uniaxial straining. Although the constitutive equations are inherently time dependent, the formulation and associated calculation procedure would reduce to time independent classical plasticity under the appropriate limiting conditions, Bodner (1968), Partom (1970).

In the analysis of Bodner and Partom, the inelastic properties of the material at a given microstructural state and temperature are taken to be completely governed by a relation between the second invariant of the plastic deformation rate, D_2^p, and the second invariant of the stress deviator J_2, i.e. $D_2^p = f(J_2,T)$ for a constant material state. The theories of Hart and of Miller are essentially the same on this point and there is a fair amount of experimental evidence to support the existence of this postulated relationship. However, there are important differences in the methods used by the various investigators for representing and treating this function and on other aspects of the formulation. For example, Hart introduces an "anelastic strain" term as a state variable to account for various transient effects, while Bodner and Partom introduce an "anelastic stress", which is not a state quantity, to represent certain visco-elastic effects such as internal damping and creep transients.

A basic assumption in the theories of Bodner and Partom and of Hart is that the functional relation $D_2^p = f(J_2, T)$ depends in a unique manner on a single state variable referred to as the "hardness". This quantity, termed Z in the present discussion, represents the overall microstructural state of the material with respect to its resistance to plastic flow. For most materials and conditions, D_2^p would decrease with increasing Z at constant stress, Fig. 1, although counter examples are possible over limited ranges and are admissible in the general formulation. "Hardness" defined in this manner has no obvious physical identity although it has characteristics similar to the stored energy of cold work (SECW) which is the elastic energy of the internal microstructural arrangement.

On the basis of these properties, the hardness variable Z can be incorporated into the equation for D_2^p so that it could be expressed as $D_2^p = F_1(J_2, Z, T)$. The essential problem then is to determine the evolutionary equations for Z for general loading histories. This paper is concerned with developing a general procedure for calculating Z that is intended to apply for all loading histories including reversed loading and nonproportional loading paths in stress space. Such a procedure would be somewhat analogous to a scheme for determining the changes in shape and size of yield surfaces consequent to arbitrary loading histories, i.e. it would correspond to a work-hardening model. Various proposed work-hardening models have been analyzed and discussed recently by Hunsaker, et al. (1973) in relation to their representation capabilities. The test of any procedure is, of course, the extent it can represent actual behavior with a minimum of material constants and auxiliary conditions.

Development of Hardness Law

The formulation of Bodner and Partom, which will be the basis of the following discussion, considers the total deformation rate $d_{ij} = 1/2(v_{i,j} + v_{j,i})$, which is identical to the strain rate for small strains, to be decomposable into elastic (reversible) and inelastic (non-reversible) components,

$$d_{ij} = d^{e}_{ij} + d^{p}_{ij} \tag{1}$$

where d^{e}_{ij} is related to the stress rate through a strain energy function or Hooke's Law. From the plastic flow law,

$$d^{p}_{ij} = \lambda\, s_{ij} \tag{2}$$

it follows that

$$D^{p}_{2} = (1/2) d^{p}_{ij} d^{p}_{ij} = (1/2)\lambda^2 s_{ij} s_{ij} = \lambda^2 J_2 \tag{3}$$

Assuming that

$$D^{p}_{2} = F_1(J_2, Z, T) \tag{4}$$

as discussed in the Introduction, d^{p}_{ij} can then be expressed in terms of state variables, namely

$$d^{p}_{ij} = [F_1(J_2, Z, T)/J_2]^{1/2} s_{ij} \tag{5}$$

A particular form adopted for (4) motivated by expressions used for the relation between dislocation velocity and stress is

$$D^{p}_{2} = D^{2}_{0} \exp[-\tfrac{1}{3}\, Z^2)^{n} (\tfrac{n+1}{n}) / J^{n}_{2}] \tag{6}$$

where Z is the hardness parameter, D_0 is a scale factor, and n is a material constant that directly influences the slope of the curves on the D_2^p, J_2 plane and therefore the rate sensitivity (Fig. 1). Once a procedure for calculating Z for arbitrary loading histories is adopted, the equations could be incorporated into problems of structural analysis and are amenable to numerical solution on a step by step basis. Techniques for solving structural problems based on these constitutive equations were demonstrated by Partom (1970) and Bodner and Partom (1972a) using the finite difference method, and by Newman et al. (1976) using the finite element method.

For uniaxial stress states of constant stress sign, the hardness parameter Z was taken to be an increasing function of plastic work W_p. The particular form adopted for the calculations was

$$Z = Z_1 + (Z_0 - Z_1)\exp(-m'W_p) \tag{7}$$

which is shown graphically in Fig. 2 by curve A. The material constants Z_0, Z_1, m', and also D_0 and n could be obtained for a given material on the basis that two stress-strain curves at different strain rates would be adequately predicted by the equations. These constitutive equations have been shown by Bodner and Partom (1975) to accurately represent Ti subjected to various uniaxial loading histories: rapid changes of strain rate, loading and unloading, creep, and stress relaxation. Recent work has also shown their applicability for strongly strain hardening materials, Cu and Al, for very rapid changes of strain rate. Plastic work is a convenient quantity to use as the measure of hardness as it is scalar and isotropic and has some of the properties of a state variable. It is noted that the choice of plastic strain as the hardness measure would not lead to correct predictions for many loading conditions, e.g. changing strain rates, stress relaxation.

Taking $Z = Z(W_p)$, (7), corresponds to isotropic hardening and therefore would not hold for reversed uniaxial stressing or for general non-proportional loading. A scheme to calculate changes in Z due to reversal of stress sign, as for uniaxial cyclic loading, has been developed by Bodner and Partom (1976) based on physical considerations and the concept of kinematic hardening. It involves reducing the hardness level of the material from the value Z acquired by the past loading history of constant stress sign to a new value, Z_0', where

$$Z_0' = Z_0 - \Delta Z = 2Z_0 - Z \geqslant 0 \tag{8}$$

and where

$$\Delta Z = Z - Z_0 \tag{9}$$

That is, the new Z_0 to be used in (7) after stress sign change would be the original Z_0 minus the difference between the value of Z acquired during the preceding half cycle of loading and Z_0. This would lead to curve B shown in Fig. 2 where state p moves to p'. It is noted that Z_0' is required to be always positive for the hardness concept to have physical meaning. The value of W_p is reset to zero for subsequent calculations of Z. This scheme implies that the hardness gain in one direction with respect to the original hardness, ΔZ, is reduced by an equal amount in the opposing direction, which is equivalent to introducing the concept of kinematic hardening within an overall framework of isotropic hardening. Further deformation with constant stress sign would again correspond to isotropic hardening behavior on the basis of (7). It is noted that ΔZ could be negative which would occur due to a small excursion into the reversed stress state and subsequent return. In such a case, the procedure would lead to curve C in Fig. 2 becoming applicable ($p \to p''$). Other terms in (7) that could be influenced by a change in stress sign are m' and Z_1. It is reasonable to keep m', the hardening rate, constant but Z_1 may have to be altered in a manner similar to Z_0 to account for cyclic hardening or softening, i.e. changing Z_1 to a new $Z_1' = Z_1 + q\Delta Z$ where q is a new material constant.

This procedure avoids the introduction of a "back stress" of varying sign which is, in effect, an artificial term to account for variations in resistance to inelastic deformation in different directions (plastic anisotropy). Altering the hardness parameter seems to be more direct and closer to the physical situation. The only additional constant that might be introduced is q. For titanium, it was found that q = 0 was adequate for cyclic loading, but a non-zero q is probably required for materials that exhibit strong cyclic hardening or softening. It was noted by Bodner and Partom (1976) that the suggested discontinuous changes in Z do correspond closely to the very rapid changes in SECW observed upon stress reversal by Halford (1966). This seems to be a consequence of stress reversal initially causing a partial breakup of the microstructural arrangement developed by straining in one direction into a more disordered state.

The constitutive equations modified in the above manner lead to reasonably representative cyclic stress-strain curves except for a sharp "knee" that exists regardless of the particular scheme adopted to regulate the hardness parameter. This implies that factors other than changes in hardness are important at the onset of reversed loading. To account for an increased plastic deformation rate upon sign changes, a second term was added to d^p_{ij} (5),

$$d^p_{ij} = [F_1(J_2,Z,T)/J_2]^{1/2}s_{ij} + \omega s_{ij} \tag{10}$$

The physical motivation for this additional term is an apparent increase in dislocation mobility for small strains in the reversed direction. The initial value of ω in (10) was taken equal to $C(\delta Z)$ where C is a constant and δZ is the increment in Z accumulated in the immediate past loading history of constant stress sign minus the Z accumulated subsequent to the sign change. This softening effect therefore decreases in importance with increasing deformation and eventually $\omega \to 0 (\omega \geq 0)$. The hardening law would therefore again approach the isotropic case unless further changes in stress sign occur, which seems to be in agreement with experimental observations of a number of investigators reported by Saczalski and Stricklin (1975) in an ONR Plasticity Workshop.

Adoption of both the change in hardness and the addition of the s_{ij} term was found by Bodner and Partom (1976) to lead to fairly good predictions of cyclic loading behavior. It is also possible to consider arbitrary combinations of cyclic and unidirectional loading circumstances by the proposed method. Under certain circumstances, such as at high temperatures, it will also be necessary to introduce additional equations to account for changes in Z due to annealing, strain ageing, or other metallurgical effects. These equations would have the form

$$\dot{Z} = F_2(Z, J_2, T) \tag{11}$$

and would operate in conjunction with the other equations and procedures.

The next step is to consider varying multidimensional states of stress including changes in sign of the stress components. This could be achieved by generalizing the procedure outlined for uniaxial reversed loading such that that condition would be a special case. To do this, it is desirable to introduce a characteristic direction associated with the current stress tensor with respect to its influence on plastic deformation. This characteristic direction could be defined as that of the resultant stress vector in stress space or the normal vector to the plane of maximum shearing stress in real space. Changes in these directions with each load increment could then be calculated since an incremental procedure is used for both continuous and discontinuous loading histories. Calculation of the angular changes based on either of the above definitions would be tedious, however, for general multiaxial stress states. An alternative definition of the characteristic direction is that of the plastic deformation rate vector in deformation space. Using the flow law (2), the angular change, α, of the characteristic direction is then expressed by

$$\alpha_{mn} = \cos^{-1}\left[\frac{1}{2}\,\frac{(s_{ij})_m\,(s_{ij})_n}{(J_2)_m^{1/2}(J_2)_n^{1/2}}\right] \tag{12}$$

where the subscripts m and n refer respectively to the states just prior to and subsequent to the load increment. Eq. (12) is relatively simple to compute at each loading stage.

Anisotropy of the hardness parameter Z could then be specified as a function of the angle α from the current characteristic direction. A simple and physically reasonable form of such a law is for the hardness accumulated from the initial state Z_0, i.e.

$$(\Delta Z)_m = Z_m - Z_0 \tag{13}$$

to vary in deformation space as the cosine of the angle α where Z_m is the current hardness value in the characteristic direction associated with the stress tensor $(\sigma_{ij})_m$. Consequent to a further increment in loading that would bring the stress state to $(\sigma_{ij})_n$, the new hardness value Z_n would become

$$Z_n = Z_0 + (\Delta Z)_m \cos\alpha_{mn} + (\Delta Z)_{mn} \tag{14}$$

The first two terms in (14) correspond to the state m hardness value in the direction associated with state n. The hardness addition $(\Delta Z)_{mn}$ due to the magnitude of the load increment would be computed from (7) in which Z_0 is replaced by Z_0' where

$$Z_0' = Z_0 + (\Delta Z)_m \cos\alpha_{mn} \tag{15}$$

and W_p is reset to zero at each load increment. Utilizing (7) in such an incremental manner rather than treating $Z = Z(W_p)$ as a continuous function would lead to slightly different hardening characteristics for the same material constants. An illustration of this effect for unidirectional loading ($\alpha = 0$) is shown by the difference in the shape of curves A and C in Fig. 2 for $Z > Z_0''$. The other constants in (7), m' and Z_1, could be varied if necessary according to some scheme with each load increment, but this is probably unnecessary for most materials and conditions (except for strong cyclic hardening or softening).

It is obvious that except for the slight change in using (7) in an incremental rather than total manner, the proposed method would reduce to that of the earlier work for the unidirectional stress case $\alpha = 0$. It does reduce exactly to the method described previously for uniaxial cyclic loading for which $\alpha = \pi$ immediately upon change of the stress sign. The method would also lead to no cross effects since hardening in one direction has no influence in directions for which $\alpha = \pi/2$.

In addition to accounting for the change in hardness due to changes in stress direction, it would also seem necessary to generalize (10) for multiaxial loading. The parameter ω in (10) is, however, a derived quantity brought about by hardness changes so that further explicit variations of ω with α would not be required. The α variation is incorporated into the hardness parameter on which ω depends. The softening effect represented by ω stems essentially from the lowering of hardness values below the original state due to stress changes. An alternative definition of ω, compared to the previous one, which would be applicable for general multiaxial loading paths would be for ω to be the difference between the original hardness state Z_0 and the current state Z_m, i.e.

$$\omega_m = Z_0 - Z_m = -(\Delta Z)_m \geq 0 \qquad (16)$$

By this definition, only conditions that lead to a lowering of the hardness from its original value would result in a non-zero ω term. This could occur only if total changes in α greater than $\pi/2$ take place in the loading history. The calculation procedure for ω would agree with that described previously for simple uniaxial reversed loading histories, e.g. equal strain reversals, but could differ from it for other cases. This method, i.e. (10) and (16), for including additional inelastic effects upon stress reversals seems to be physically reasonable and is relatvely simple to use. Evaluation of its general validity depends, of course, on comparisons of predicted and experimental results.

Consequences of Hardening Law

The proposed law indicates that the hardening effect is strongest in the direction of the plastic deformation increment vector and varies as the cosine of the angle in other directions in deformation space. It therefore leads to a hardness "bulge" forming in the deformation direction. Since the hardening law is continuous, it does not admit the concept of "corners".

A complete reversal of straining direction would result in an initial softening equal to the previous hardening. This effect diminishes with continued deformation and the hardening approaches the isotropic condition as long as no further changes in straining direction occur. In general, large straining in a single direction would lead to isotropic type hardening regardless of prior history. The proposed method therefore includes "kinematic type" hardness effects for changes in straining direction within an overall framework of isotropic hardening.

The initial softening effects would diminish when the angular change in straining direction is less than π. For example, softening subsequent to an initial compressive prestrain would be less in compression in a different coordinate direction or in torsion than in tension along the initial coordinate axis. This agrees with experimental observations of Lindholm, Yeakley, and Davidson (1974) for multiaxial testing of beryllium. A further consequence of the proposed method is that cross hardening (or softening) effects would be non-existent. This results from the dependence of effective hardening on the cosine of the straining direction.

The above consequences of the proposed hardening law appear to be consistent with general observations on hardening effects under multiaxial loading reported in the papers compiled by Saczalski and Stricklin (1975) and in the review paper of Michno and Findley (1976). Most experimental studies on hardening have been concerned with relating their findings to various proposed models for the growth and distortion of yield surfaces with straining. The suggested procedure could be interpreted in a similar manner even though it was developed for constitutive equations that do not require a yield condition.

Starting with the von Mises yield criterion, the proposed method would correspond to hardening (i.e. expansion of the yield surface) taking place in the direction of plastic straining corresponding to the appplied stress state. Hardening in other directions in deformation space due to the same loading condition would vary as the cosine of the subtended angle. The yield surface in stress space would therefore both distort and translate as plastic straining proceeds. For continually turning stress paths, an incremental approach is necessary and the yield surface would be recalculated at each loading step. The calculation of the hardness increment at each step could be based on a plastic work equation, but for the case of yield surface expansion this could also be done by other methods in general use. Which particular method is used is not critical in this case since the yield surface concept has much more limited representation capability for general loading histories than that obtainable by the elastic-viscoplastic constitutive equation approach.

References

BODNER, S.R.	1968	Proc. Conf. on Mechanical Behavior of Materials under Dynamic Loads, (edited by U.S. Lindholm), p.176, Springer-Verlag, N.Y.
BODNER, S.R. and PARTOM, Y.	1972a	J. Appl. Mech., 39, 751.
	1972b	Proc. Eighth Congress of the Intern. Council of the Aero. Sciences, ICAS paper 72-28, AIAA, N.Y.
	1975	J. Appl. Mech., 42, 385.
	1976	"Constitutive Equations for Cyclic Loading of Rate Dependent Materials," MML Report 51, Technion, Haifa, Israel.
HALFORD, G.R.	1966	"Stored Energy of Cold Work Changes Induced by Cyclic Deformation," Ph.D. Thesis, T.A.M. Dept., Univ. Illinois, Urbana.
HART, E.W.	1975	Constitutive Equations in Plasticity (edited by A.S. Argon), M.I.T. Press, Cambridge, Mass.
HUNSAKER, B., VAUGHAN, D.K., STRICKLIN, J.A., and HAISLER, W.E.	1973	"A Comparison of Current Work-Hardening Models used in the Analysis of Plastic Deformations," Report TEES-RPT-2926-73-3, Texas A&M Univ.
KELLY, J.M. and GILLIS, P.P.	1974	J. Appl. Phys., 45, 1091.
LINDHOLM. U.S., YEAKLEY, L.M., and DAVIDSON, D.L.	1974	"Biaxial Strength Tests on Be and Ti Alloys," AFML-TR-74-172, Air Force Materials Lab., WPAFB, Dayton, Ohio.
MICHNO, M.J. and FINDLEY, W.M.	1976	Int. J. Non-Linear Mech. 11, 59.
MILLER, A.	1976	J. Engin. Mat. and Tech., Trans. ASME, 98, (I) 97, (II) 106.
NEWMAN, M., ZAPHIR, Z., and BODNER, S.R.	1976	J. Computers & Structures, 6, 157.
PARTOM, Y.	1967	"Numerical Solution of Continuum Mechanics Problems," MML Report 11, Technion, Haifa, Israel.
	1970	Inter. J. Non-Linear Mech., 5, 475.

RICE, J.R.	1970	J. Appl. Mech. 37, 728.
	1975	Constitutive Equations in Plasticity (edited by A.S. Argon), M.I.T. Press, Cambridge, Mass.
SACZALSKI, K.J. and STRICKLIN, J.A. (editors)	1975	Proc., ONR Plasticity Workshop, Texas A&M Univ., RPT. 75-51.

List of Captions

Fig. 1 - Dependence of invariant of inelastic deformation rate D_2^p on stress deviator invariant J_2 for various hardened states.

Fig. 2 - Dependence of parameter Z (measure of hardened state) on plastic work showing effect of stress reversal (p to p').

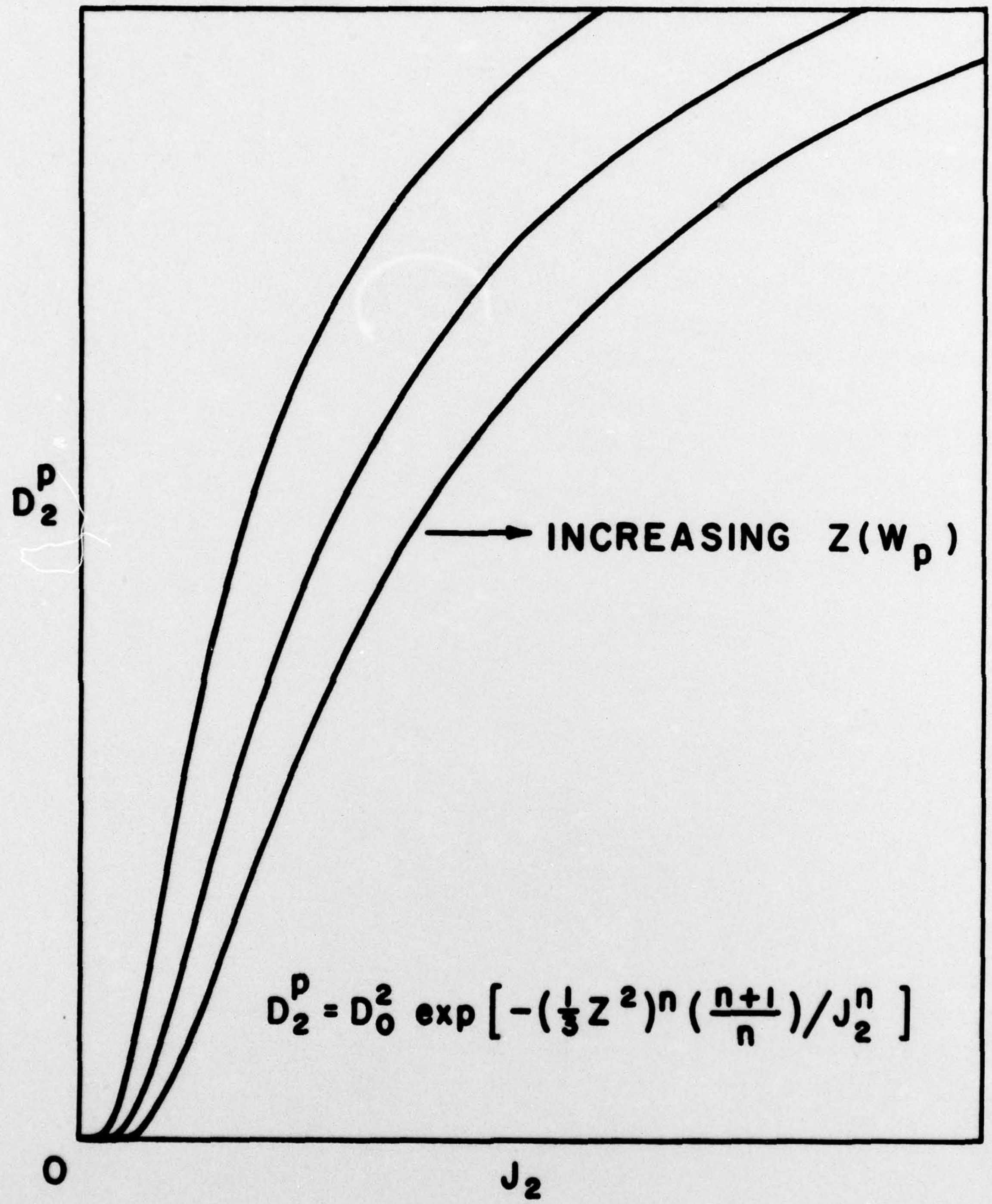
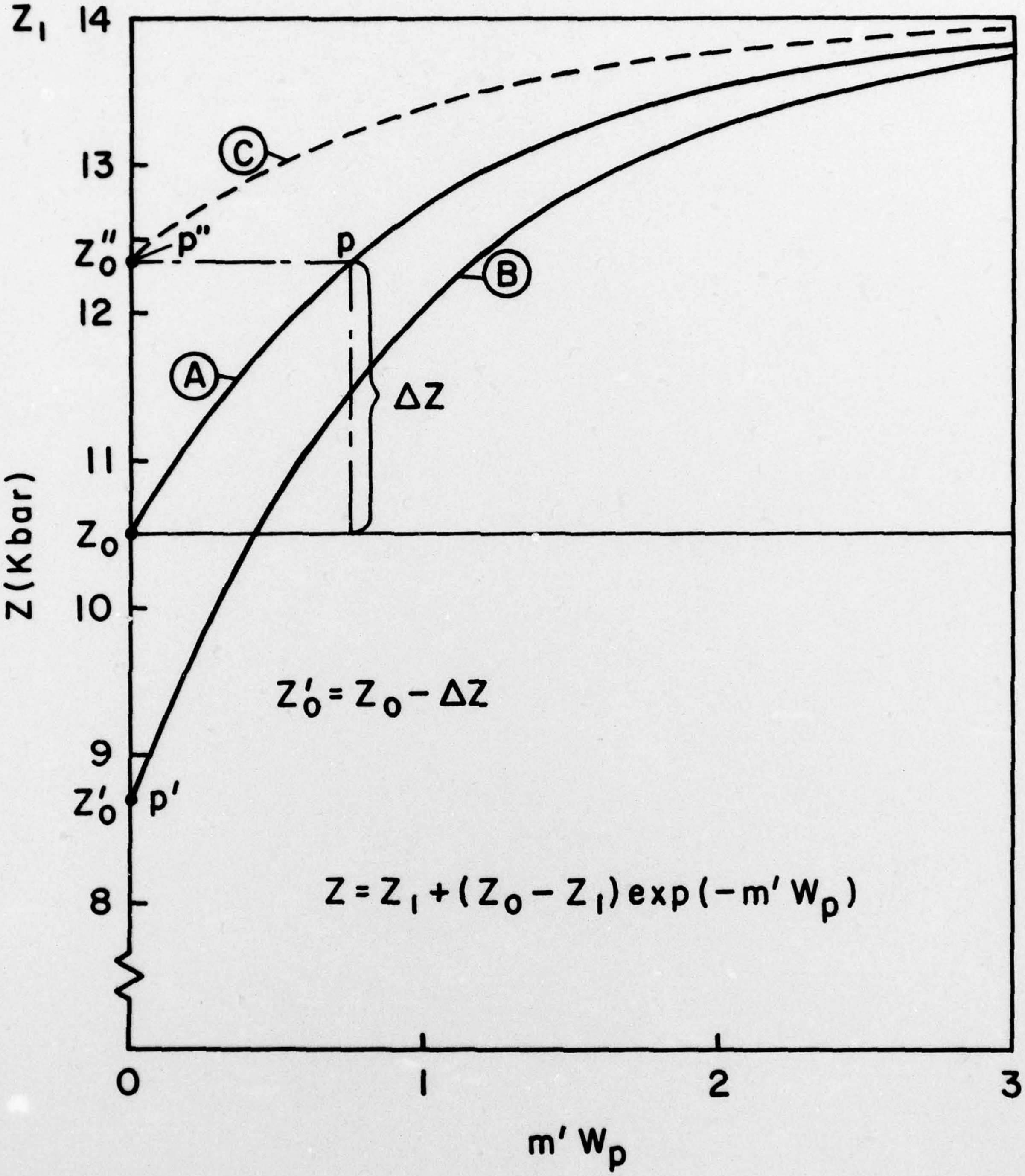

D_2^p
INCREASING $Z(W_p)$
$D_2^p = D_0^2 \exp\left[-\left(\frac{1}{3}Z^2\right)^n\left(\frac{n+1}{n}\right)/J_2^n\right]$
O
J_2

FIGURE 1



Z_1
14
13
12
11
10
9
8
Z_0''
P''
P
Z_0
Z_0'
P'
C
A
B
ΔZ
Z(Kbar)
$Z_0' = Z_0 - \Delta Z$
$Z = Z_1 + (Z_0 - Z_1)\exp(-m' W_p)$
0
1
2
3
$m' W_p$

FIGURE 2